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Working Paper No. 113

July 2016

BERG Working Paper Series

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ISBN 978-3-943153-32-3

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Strategic Sequential Voting*

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This version: July 27, 2016

Abstract

In this paper, we study the potential implications of a novel yet natural voting system: *strategic sequential voting*. Each voter has one vote and can choose when to cast his vote. After each voting period, the current count of votes is publicized enabling subsequent voters to use this information. Given the complexity of the general model, in this paper we study a simplified two-period setting. We find that, in elections involving three or more candidates, voters with a strong preference for one particular candidate have a strategic incentive to vote in an early period to signal that candidate's viability. Voters who are more interested in preventing a particular candidate from winning have an incentive to vote in a later period, when they will be better able to tell which other candidate will most likely beat the one they dislike. *Strategic sequential voting* may therefore result in voters coordinating their choices, mitigating the problem of a Condorcet loser winning an election due to mis-coordination. Furthermore, a (relatively) strong intensity of preferences for the preferred candidate can be partially expressed by voting early, possibly swaying the choice of remaining voters.

JEL-Classification: D72, D71, C72

Keywords: sequential voting, elections, endogenous timing, strategic timing

*We thank Nemanja Antic, David Austen-Smith, Eddie Dekel, Georgy Egorov, Péter Eső, Timothy Feddersen, Johannes Hörner, Ehud Kalai, Peter Klibanoff, Christoph Kuzmics, Nicola Persico, Marco Sahm, Eran Shmaya, Jörg Spenkuch, Stefanie Schmitt, and participants at a seminar at the Northwestern University, at SING10 and at economic theory workshops at the University of Bamberg for their helpful comments and suggestions. Florian Herold gratefully acknowledges the support received from the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme (FP7/2007-2013) under REA grant agreement PCIG11-GA-2012-322253. Julio González-Díaz acknowledges support from the Spanish Ministry of Economy through project MTM2014-60191-JIN and from Xunta de Galicia through project EM 2012/111.

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1 Introduction

It is a well-known fact that single round ballots in which the winner is chosen by simple plurality from more than two candidates can create problems. In particular, a Condorcet loser, who would lose the election to any other candidate in a pairwise election, may win in simple plurality voting if the voters who prefer another candidate fail to coordinate their votes in favor of one particular contender. Some electoral systems attempt to mitigate this effect by having several voting rounds, with a run-off election between the most successful candidates (e.g. the presidential election in France). In practice, multi-round ballots may be very expensive. Not only is it expensive to organize the ballots; electoral campaigns are also costly and time-consuming. Importantly, voter turnout may decrease rapidly as the number of rounds increases.

In this paper we put forward and analyze a different sequential electoral system. In this system, each voter is free to decide when to cast his[1] vote over a certain period of time and each candidate's intermediate score (number of votes) is publicized in the course of the multi-round ballot. More precisely, the ballot is divided into a fixed number of periods and the intermediate score is announced after each period. This way, late voters can vote contingent on the scores at that point. We focus our analysis on the case of a two-period ballot. In practice, polling places could be open all day, and an announcement is made at noon stating how many votes have been cast for each candidate by then. Indeed, modern information technology makes it easy to have more than two voting periods or to update the score with every vote cast. The cost of organizing such an election is almost equal to that incurred for a single-round ballot. This voting system could be used for large electorates, but may also be of interest for relatively small groups or committees. Arguably, online tools such as Doodle or informal votes by email using the "reply to all" option already have a similar structure.

This sequential voting system, while respecting the "one person, one vote" principle, has a couple of interesting features: i) it allows the relative strength of preferences over candidates to be partially expressed by the choice of timing and ii) it mitigates the potential problem of a Condorcet loser winning an election due to mis-coordination. The strategic

[1]We use female pronouns for candidates and male pronouns for voters to enable a distinction to be made between voters and candidates.

richness of this sequential setting arises from the tension between two conflicting interests: voting early to make your preferred candidate look stronger versus voting late to make a more informed decision. Our analysis shows that voters who care most about preventing a certain candidate from winning (averters) have an incentive to wait until the intermediate score reveals which other candidate is most likely to win. In contrast, voters who support most strongly a certain candidate (partisans), have an incentive to vote early in order to signal their favorite candidate's competitiveness.

It is worth noting that the analysis is restricted to the two-period case to ensure the model remains tractable. Nonetheless, we believe that the aforementioned insights into the behavior of partisans and averters, and the implications concerning coordination remain valid as the number of periods increases. Indeed, we would expect voter coordination to increase in line with the number of periods.

Interestingly, the results obtained for our game-theoretic model deliver a number of testable implications. These include voters being split into the two periods depending on their preference intensity, and the reduced likelihood of a Condorcet loser to win due to more vigorous voter coordination. Another testable result we find is that the stronger the Condorcet loser is ex-ante, the fewer people vote in Period 1. This is natural because greater voter coordination is required to prevent such a candidate from winning. Although the empirical analysis of these insights is most certainly an important path for future research, it is beyond the scope of this paper.

The remainder of our paper is organized as follows. In the next subsection, we briefly discuss the related literature. In Section 2, we describe the formal model and define key terms for the ensuing analysis. We also derive some general results, demonstrating that in the relevant equilibria voters have threshold strategies: partisans who value their intermediate candidate below the threshold vote in the first period while averters who value their intermediate candidate above the threshold, vote in the second period. A complete analysis of all equilibria in this general setup does not appear to be a realistic goal because, as in most sequential voting models, the pivotal analysis soon becomes very complex, resulting in a plethora of equilibria. We therefore limit our attention to two specific setups, each chosen to highlight one key effect of strategic sequential voting. For both setups we present a theoretical and a numerical subsection. In Section 3, we consider a very symmetric setup in which all candidates are ex-ante equally strong; we consider the case where voters are either complete partisans (they only care about one candidate) or complete averters (they only care about preventing one of the candidates from winning). The setup helps us to illustrate how the strength of preferences influences in which period voters cast their vote, improving welfare relative to simultaneous plurality voting. In Section 4, we focus on how strategic sequential voting can facilitate voter coordination and prevent the victory of a Condorcet loser, i.e. a candidate who is the least preferred choice for the majority of voters. To this end, we investigate a partially deterministic setup in which one candidate is known to be a Condorcet loser, but voters still need to coordinate their votes in favor of another candidate to prevent the Condorcet loser from winning. In Section 5, we conclude with a discussion of our results and outline open questions for further research.

1.1 Related literature

From the large literature on voting we focus on the work most related to our setting. From the literature on simultaneous voting Myerson and Weber (1993), Myatt (2007), and implicitly also Palfrey (1989) consider the coordination problem between voters who want to prevent a Condorcet-loser from winning and are thereby related to our setup in Section 4. These papers study the implications of their results with respect to Duverger's Law, which roughly states that plurality rule leads to a two party system.[2] We discuss the relation to our work in Section 4.

Only a relatively small part of the literature on voting considers sequential voting, and typically either all voters can cast a vote in all periods or voters can not choose when to cast their vote. The papers most closely related to our approach are probably those by Dekel and Piccione. In Dekel and Piccione (2000), symmetric binary elections with only two candidates are considered. They show that the symmetric equilibria of the simultaneous voting game are also equilibria of the sequential voting game.[3] However, the effects that interest us do not occur in this setting involving only two candidates.

In Dekel and Piccione (2014), three candidates are considered. Although their setup is similar to ours and they also allow voters to choose when to vote, there is one key difference.

[2]Myatt (2007), for instance, refers to Duverger (1954).

[3]Battaglini (2005) shows that, with abstention and costly voting, the set of simultaneous voting and sequential voting can be disjoint.

In contrast to our setup, they consider situations in which voters do not yet know their preferences over candidates at the time of deciding in which period they want to cast their vote. This assumption is realistic for the situations that interest them, such as the US presidential primaries where each state has to choose the timing of the ballot without even knowing the contenders. We are interested in a voting system where voters decide their timing on the day they cast their vote. For our purpose, therefore, it is more realistic for preferences to be known when the timing decision is made. This difference in setup is important for our key finding that partisans tend to vote early and averters tend to vote late, which hinges on the assumption that voters differ in their relative intensity of preferences over candidates at the time of deciding when to cast their vote.

A second key difference to our approach is that the analysis in Dekel and Piccione (2014) mainly concentrates on what they call *persistent strategies*, in which a second-period voter continues to vote for his most preferred candidate as long as the candidate has a positive probability of winning the election. One of the central results in their analysis is that, if all voters are restricted to persistent strategies, then voting for one's favorite candidate in the first period weakly decreases the chance of this candidate winning relative to voting for the favorite candidate in the second period. This implies that, if voters are restricted to persistent strategies, all equilibria are equivalent in outcome to simultaneous voting.[4] On the other hand, if the sets of strategies are not restricted to persistent ones, then non-persistent strategies may be needed to obtain an equilibrium. In particular, they develop a special model, called the *x-model*. They use this model to show that, if it is ex-ante known that voters value their second-favorite candidate sufficiently close to their favorite candidate, then the following holds in every equilibrium: i) no voter uses persistent strategies and ii) the probability that everybody will vote in the same period is bounded away from zero.[5] The analysis in the present paper focuses precisely on equilibria involving non-persistent strategies, which we call *responsive*, and the resulting strategic aspects of sequential voting.

Several papers consider sequential voting with an exogenously given order. Callander (2007) considers bandwagons and momentum in sequential voting with two candidates under incomplete and asymmetric information and compares the outcome with the equilibrium

[4]Refer to Theorem 1 in Dekel and Piccione (2014).

[5]Refer to Theorem 2 and Corollary 1 in Dekel and Piccione (2014) and to Lemma 2 of this paper.

when voting is simultaneous.[6] Morton and Williams (1999) theoretically and empirically compare sequential voting elections with simultaneous ones. Bag, Sabourian, and Winter (2009) consider sequential elections where one candidate is eliminated in each round. Hummel (2012) considers sequential elections involving three candidates where voters have perfect information about their private preferences, but do not know the distribution from which the other voters' preferences are drawn. Half of the voters cast their vote in the first period and the other half in the second period in an exogenously given order. Second-period voters have an incentive to stop voting for the candidate who comes last in the first round. Battaglini, Morton, and Palfrey (2007) compare simultaneous and sequential elections with two candidates, when voting is costly and information is incomplete in a common interest election. Deltas and Polborn (2012) consider the effect of candidate withdrawal in the sequential US presidential primary elections. Deltas, Herrera, and Polborn (2015) consider the tradeoff between voter coordination and learning about a candidate's quality. They find that sequential voting minimizes vote splitting (several candidates competing for the same policy position) in late districts, but voters may coordinate their votes in favor of a low-quality candidate. Hummel and Holden (2014) consider the optimal ordering of primaries with two candidates of different quality from a social planner's perspective.

There is also a partially related literature on how pre-election polls can serve as a coordination device, for instance Andonie and Kuzmics (2012), Fey (1997), and Hummel (2014). However, the incentives in pre-election polls are different to those in our setting, since a voter can support one candidate in a pre-election poll, but switch and vote for another candidate in the real election (in our setting, Period 1 votes are binding).

2 The Benchmark Model

As argued in the introduction, although we would ideally like to study models with an arbitrary number of candidates and voting periods, the complexity of the whole sequential voting setting calls for a significant simplification of the model.

Throughout the paper we therefore consider an election with three candidates (or alternatives), A, B, and C, and $N \geq 4$ voters. The voting procedure has to select exactly one

[6] Momentum and herding behavior in sequential elections is also considered in Ali and Kartik (2012), Fey (1997), Wit (1999), Morton, Muller, Page, and Torgler (2015), and Knight and Schiff (2010).

of the candidates using the simple plurality rule, i.e. every voter can cast one vote and the candidate with the highest number of votes is elected. Whenever there is a tie, the winner is chosen randomly, with all candidates in the tie being equally likely to win.

The main departure from the existing literature is that the election is sequential, consisting of two periods. Each voter can strategically decide to vote in either Period 1 or Period 2. In the latter case, he would know how many votes each candidate received in Period 1, which we call the *score*.

The type of voter is given by the utility he attaches to each candidate being elected. We assume, without loss of generality, that these utilities have been normalized so that each voter i attaches utility one to his most preferred candidate, utility zero to his least preferred one, and utility $v_i \in [0, 1]$ to his intermediate candidate. Thus, if we let Π denote the set of possible orderings of $\{A, B, C\}$, the type of voter i consists of two elements: i) an ordering $\pi \in \Pi$ of the three candidates and ii) utility v_i attached to his intermediate candidate. We commonly refer to voters with a low v_i as *partisans* and voters with high values of v_i as *averters*, since they want to avert victory of a certain candidate, but like the other two. For the sake of exposition, we say that a voter is an AB-voter, for instance, meaning that A is his preferred candidate and B his intermediate one.

For the time being, we assume that types are drawn *i.i.d.* from a certain probability distribution, before the election starts, i.e. knowledge of the valuation of a group of voters provides no new information about the remaining voters' preferences. Thus, we focus on the case of private values and abstract from any considerations about the information aggregation provided by elections.

Definition 1. Two candidates are (ex-ante) *symmetric* if the distribution of probability from which types are drawn treats them identically.

2.1 Strategies

Given a voter i, a (possibly mixed) strategy σ_i specifies, for each possible type, what i's behavior would be given that type. More precisely, it specifies for every possible type the probability of i voting in Period 1 and the probabilities with which he would chose each candidate if voting in Period 1 and at each possible score after Period 1. We denote strategy profiles by σ.

Definition 2. A strategy profile is *symmetric* if all voters of the same type follow the same (possibly mixed) strategy.

When working with symmetric profiles, one simply needs to specify the behavior of each possible type of voter. For most of the analysis in this paper, we concentrate on equilibria in which the voters' strategies are symmetric.[7]

We now introduce an anonymity property, which requires that the strategies treat symmetric candidates identically. Although the idea is standard, the formalization in this setting is rather cumbersome. Note that the only information provided to a voter during the election, apart from his own type, is the election score after Period 1.

Definition 3. A strategy profile σ is *anonymous* if, for each voter i, each pair of symmetric candidates, say D_1 and D_2, and each pair of types θ and θ' that only differ in that the roles of D_1 and D_2 have been interchanged, the following holds for σ_i:

> If under type θ, at a given moment of the election and given voter i's information, he votes for candidate D_1 with probability p, then, under type θ', at an analogous moment in which the information about D_1 and D_2 has been interchanged, voter i will vote for candidate D_2 with the same probability p.

The following property merely captures the natural feature that voters in Period 2 may be attracted towards stronger candidates (reducing the probability of "wasting" their vote).

Definition 4. A strategy profile is *weakly monotonic* if, for each candidate D, once we fix the number of votes in Period 1 for the other candidates, the expected total share of votes for D at the end of the election is weakly increasing in the number of votes she gets in Period 1.

A crucial aspect of this paper is the need to understand the extend to which voters in Period 2 are influenced by the score revealed after Period 1. The next two definitions capture two extreme degrees of responsiveness or unresponsiveness.

Definition 5. A strategy profile is *unresponsive* if, for each candidate D and each voter i, no deviation of i changes D's expected total share of votes at the end of the election beyond voter i's vote.

[7]As usual, symmetry can be broken when studying deviations, so the equilibria in symmetric strategies are not weaker than asymmetric ones.

Definition 6. A strategy profile is *fully responsive* if it is weakly monotonic and, moreover, in Period 2 a voter votes for the candidate who is leading (if any) from the candidates who give him a positive utility.

Full responsiveness is a very strong form of monotonicity in which voters react by voting for the candidate who seems stronger after Period 1 (provided they receive some positive utility if she wins). Although this extreme form of monotonicity may not be appealing in general, we will present two settings in which full responsiveness is natural. It is also worth noting that, under some circumstances, fully responsive strategies can be incompatible with equilibrium conditions. We illustrate this in the following example.

Example 1. Consider a situation in which we have 100 voters, 50 of whom voted for B and 49 of whom voted for A in Period 1. Suppose, moreover, that voter i is the remaining voter and his favorite candidate is A and his second favorite candidate is B with utility $v_i \in (0,1)$. Then full responsiveness would require that i votes for B, but he would get a higher expected utility by voting for A. $\Diamond$

Situations like the one described in Example 1, where a voter knows after Period 1 that he is the last voter and that his vote will make a difference, are very unlikely, but they can make the analysis very cumbersome without adding much insight.[8]

One of the most challenging aspects of equilibrium analysis in voting models is that the resulting pivotal calculations soon become very intricate and difficult to handle. For this reason, in Sections 3 and 4 we work with two particular cases of our model under which fully responsive strategies can be supported in equilibrium. This significantly simplifies the analysis since second-period behavior is usually pinned down.[9]

2.2 Features of the model

We now informally discuss some of the main features of our sequential voting setting, which will be formally analyzed in the rest of the paper.

[8]This is not merely a problem of having the number of voters N fixed since, even if N is drawn randomly, there will always be (probably very unlikely) realizations where a voter is almost certain that he is the last voter left and then he may essentially face the same kind of trade-off we have just described.

[9]Arguably, even in the general setting in which fully responsive strategies cannot be supported in equilibrium, they can be seen as a good approximation of real-life behavior, since they are still optimal after most histories and approximate best responses after some very unlikely ones. More importantly, we conjecture that the optimal strategies in these settings, while complex to describe precisely, would preserve the qualitative features of the ones we obtain for the simplified settings.

First, quite generally, there will be informed voting in equilibrium and in both of the two periods some voters will cast their vote. The intuition is simple. On the one hand, if I know that everybody else will vote in Period 1, then I would prefer to wait until Period 2 to make an informed decision. On the other hand, if I know that everybody else will vote in Period 2, then I would have to vote without further information in any case. I may then prefer to vote in Period 1 in order to influence other voters' behavior.

The above argument highlights the main incentive that we endeavor to shed light on in this paper: the trade-off between i) voting in Period 1 in order to make the preferred candidate look stronger and encourage others to vote in her favor and ii) voting in Period 2 to make a more informed decision. To illustrate this, think of an AB-voter under fully responsive strategies:

i) By voting for A in Period 1, an AB's vote mainly makes a difference if it breaks a tie between A and another candidate (increasing coordination on A) or it induces a tie (increasing the coordination on A and reducing the coordination on the candidate who tied with A).

ii) By voting in Period 2, an AB-voter can make a difference if B is ahead of A after Period 1 and B and C are very even, so an additional vote for B can tip the election in B's favor.

Point i) is the "make your candidate look stronger" effect and point ii) is the "avoid wasting your vote" effect. In this paper, we seek to understand how these two effects come into play. This suggests a natural implication of our setting: the more partisan a voter is, the more important the first effect will be for him and the earlier he will tend to vote.

More importantly for our model, once there is informed voting in equilibrium, there is room for studying the extent to which this can lead to enough coordination to significantly decrease the chances of a Condorcet loser winning the election.

Next we formally present some relatively general properties of best responses and equilibrium strategies when we have ex-ante symmetric candidates, which already shed some light on the kind of equilibria that may arise in our setting.

2.3 Best responses and equilibria with ex-ante symmetric candidates

In this section, we explore the implications of anonymity and weak monotonicity in our sequential election model when all the candidates are ex-ante symmetric. To start with, we present a technical result that will be useful in the ensuing analysis.

Lemma 1. *Suppose that we are in a situation where the score after Period 1 is such that Candidate A is ahead of Candidate B. Further, suppose that the remaining voters are expected to vote, independently, for each candidate $D \in \{A, B, C\}$ with probability p_D, where $p_A \geq p_B$. Consider the following possible events after the end of the election:*

Event 1a. *Candidate C obtains the most votes and A is one vote behind C, with B having fewer votes than A.*

Event 1b. *Candidates C and A obtain the most votes and B is exactly one vote behind them.*

Event 1c. *Candidates C and A obtain the most votes and B is more than one vote behind them.*

Events 2a, 2b, and 2c. *Analogous to the above events but interchanging the roles of A and B.*

Then the probabilities of Events 1a, 1b, and 1c are weakly larger than the probabilities of Events 2a, 2b, and 2c, respectively. If an event has a positive probability, then the corresponding inequality is strict. This lemma also holds for all permutations of the roles of A, B, and C.

Proof. We explicitly compare Event 1a and Event 2a, with the other two cases being analogous. For the sake of exposition, suppose that there are M remaining voters who vote independently of each other and are ordered $1, 2, \ldots, M$. Suppose also that their votes are counted sequentially in this order. We represent each possible distribution of Period 2 votes with a vector $s = (D_1, D_2, \ldots, D_M)$, where D_i corresponds with the candidate chosen by voter i.

Suppose now that we are in a realization s of votes that corresponds with Event 2a, that is, Candidate C has obtained the most votes and B is one vote behind her, with A having fewer votes than B. Since voting in Period 2 started with A ahead of B, if we count the votes sequentially, there will be a voter i such that, by casting his vote, B ties with A (for the first time). Now, to realization s we associate another one, s', in which, from voter $i+1$ onwards (including him), we interchange the votes cast for A and B. As a result s' corresponds with Event 1a. Moreover, since according to s, from voter $i+1$ onwards B obtained more votes than A and $p_A \geq p_B$, realization s' is at least as likely to occur as realization s. Finally, note that if the event is realized with a positive probability, then there are other realizations with a positive probability in which B never catches up with A, who ends up just one vote behind C; this therefore corresponds with Event 1a. Combining the above arguments, if one of the events has a positive probability, Event 1a has a strictly larger probability than Event 2a. $\square$

As argued above, quite generally there will be no equilibria in which everybody votes in the same period. One exception would be a setting in which all voters attach utility 0 to their intermediate candidate, i.e. they represent truly loyal partisans for whom a best response is always to vote for their preferred candidate, and having all of them vote in Period 1 or all of them vote in Period 2 would be an equilibrium. In the next lemma, we impose an assumption that rules out this possibility.

Lemma 2. *Suppose that all candidates are ex-ante symmetric and that there is $\varepsilon > 0$ such that the interval $(1-\varepsilon, 1]$ is contained in the support of distribution F from which v_i types are drawn. Then there is no perfect Bayesian equilibrium in anonymous and symmetric strategies in which all voters vote with certainty in Period 1. Further, if the strategies are also weakly monotonic, there is no perfect Bayesian equilibrium in which all voters vote in Period 2. In particular, the result holds if F has full support on $[0,1]$.*

Proof. First, suppose that all voters vote with probability one in Period 1. Since voters' types are generated independently and all candidates are ex-ante symmetric, anonymity implies that all possible scores after Period 1 have a positive probability: given two candidates, say A and B, for each type that would vote for A we can find an (ex-ante) equally likely type that would vote for B.

Let us now consider an AB-voter with $v_i > 0$. Clearly, given that all other voters already cast their votes in Period 1, he strictly prefers to wait for Period 2. This is because for some scores revealed after Period 1 he may benefit if he votes for a different candidate from the one he would have chosen in Period 1. Therefore, we are not at an equilibrium.

Second, suppose that all voters vote in Period 2. By ex-ante symmetry and anonymity, each candidate's expected share of the votes in Period 2 equals $\frac{1}{3}$. Let us consider an AB-voter i again. Weak monotonicity implies that, by voting for A in Period 1, i will not reduce A's expected share of the votes in Period 2, which will then be $p_A \geq \frac{1}{3}$ and, by anonymity, $p_B = p_C \leq \frac{1}{3}$.

Let us now consider a BA-voter $j \neq i$ with $v_j = 1$ and consider the subgame after only one voter cast his vote in Period 1, where A is the chosen candidate. We claim that j strictly prefers to vote for A rather than B (clearly, voting for C in Period 2 is never a best response). To see this, we need to compute the probabilities of the situations in which voter j would be pivotal and would not be indifferent between voting for A or B. Importantly, note that the situations in which he strictly prefers to vote for A are captured by Events 1a, 1b, and 1c in Lemma 1. More precisely, in Event 1a voting for A would lead to utility $\frac{1}{2}$ and voting for B to utility 0; in Event 1b voting for A would lead to 1 and voting for B to $\frac{2}{3}$; and, finally, in Event 1c voting for A would lead to 1 and voting for B to 0. Events 2a, 2b, and 2c represent analogous situations, but where voting for B would be preferable. Hence, Lemma 1 implies that voter j is more likely to be pivotal in the situations where voting for A is preferable and thus voter j's best response would be to vote for A.

Now, since the incentives of a BA-voter j are continuous on v_j, there will be $\delta > 0$ such that, if $v_j \in (1-\delta, 1]$, then voter j strictly prefers to vote for A and, by assumption, the occurrence of types in any such interval has a positive probability. Clearly, all AB-voters will have an even greater incentive to vote for A. Finally, by symmetric arguments, some CA-voters and all AC-voters will also prefer to vote for A.

Therefore, since candidates are ex-ante symmetric, the expected share of the votes for A in Period 2, p_A, would be larger than $\frac{1}{3}$, the expected share if all votes where cast in Period 2. Hence, our initial AB-voter i would strictly prefer to vote for A in Period 1 instead of doing so in Period 2, which implies that having all voters cast their vote in Period 2 is not a perfect Bayesian equilibrium. $\square$

The next result shows that the kind of *threshold strategies* that are so common in voting models also arise naturally in our setting. Moreover, it also implies that equilibria will typically be in symmetric strategies.

Proposition 1. *Suppose that all candidates are ex-ante symmetric, and let σ be a weakly monotonic and anonymous strategy profile. Then the following statements hold:*

i) If σ is unresponsive, then either all best responses entail voting in Period 2 or all best responses entail voting for the most preferred candidate in Period 1 or voting for her in Period 2.

ii) Otherwise, there is a threshold $v^N \in [0, 1]$ such that, for a voter i who attaches utility $v_i < v^N$ to his intermediate candidate, it is a best response to vote for his preferred candidate in the first period. For a voter with $v_i > v^N$ it is a best response to vote in the second period.

Proof. Let σ be a weakly monotonic and anonymous strategy profile. Suppose, without loss of generality, that i is an AB-voter, with utility v_i for B.

Now, let q_1 denote a given candidate's expected share of the votes during Period 2, provided that voter i voted for her in Period 1. Clearly, by anonymity of the strategies, the other two candidates would split the remaining share evenly, $1 - q_1$. Anonymity also implies that q_1 is independent of the candidate chosen by i. Further, by the ex-ante symmetry of the candidates, if i does not vote in Period 1, all candidates will have an expected share of $\frac{1}{3}$. Then, by weak monotonicity of the strategies, $q_1 \geq \frac{1}{3}$.

Similarly, let p_1 denote the probability that, conditional on i voting in Period 1, his chosen candidate will win the election. Apart from the considerations above for q_1, $p_1 > \frac{1}{3}$, because of weak monotonicity and i's own vote.

Next, we make two observations which cover point i) in the statement of the proposition. Suppose that the strategy is unresponsive, i.e. $q_1 = \frac{1}{3}$, so σ is such that voter i is unable to sway the expected distribution of votes in Period 2. Now, two things can happen:

- According to σ, the probability that some voter $j \neq i$ will vote in Period 1 is zero. In this case, voter i is indifferent between voting in Period 1 or 2.

- According to σ, the probability that some voter $j \neq i$ will vote in Period 1 is not zero. Then, there is a positive probability that voter i can benefit from making an informed decision in Period 2. Since there is no benefit from voting in Period 1 (σ is unresponsive) he will strictly prefer to vote in Period 2.

Now we prove part ii). Suppose that $q_1 > \frac{1}{3}$. To study the best responses, we need to compare the results of voting in Period 1 with those of waiting until Period 2. Clearly, in case of voting in Period 1, since $p_1 > \frac{1}{3}$, i should vote for candidate A. We now compare the expected utility of voter i with three different strategies:

Strategy s_1. Voting for candidate A in Period 1.

Strategy s_{2A}. Voting for candidate A in Period 2.

Strategy s_2. Voting in Period 2 for the candidate who maximizes i's expected utility given the partial results after Period 1 and strategy profile σ. Since voting for C is weakly dominated, we can assume, without loss of generality, that voter i will never vote for C.

The corresponding expected utilities are denoted by U_1, U_{2A}, and U_2. By definition of p_1, $U_1 = p_1 + \frac{1-p_1}{2} v_i$. Under strategy s_{2A}, we have a probability $\bar{p}_1 > \frac{1}{3}$ of A winning the election. Anonymity again implies that the remaining probability is shared equally between B and C. Thus, $U_{2A} = \bar{p}_1 + \frac{1-\bar{p}_1}{2} v_i$. By weak monotonicity, $p_1 \geq \bar{p}_1$ since, apart from i's own vote, casting it in Period 1 may increase Candidate A's expected number of votes in Period 2 ($q_1 \geq \frac{1}{3}$). Therefore,

$$U_1 - U_{2A} = p_1 - \bar{p}_1 + \frac{\bar{p}_1 - p_1}{2} v_i,$$

which is weakly decreasing in v_i ($U_1 - U_{2A}$ equals 0 if $\bar{p}_1 = p_1$, which happens if $q_1 = \frac{1}{3}$).

We now turn now to the comparison between U_2 and U_{2A}. To this end, we can focus our attention on those realizations of the electorate in which voter i's vote can make a difference, and s_2 and s_{2A} prescribe different behavior. We claim now that $U_2 - U_{2A}$ is weakly increasing in v_i, since in all such cases, compared to s_{2A}, s_2 will increase the likelihood of B winning the election.

More precisely, consider the following notation: "ABC" represents the event that, *without the vote of voter* i, the three candidates would tie and *the realization after Period 1 was such that under* s_2 *Candidate* B *was chosen by voter* i; "AB" would represent a similar event in which Candidates A and B tie and C is more than one vote behind; "$BC > A$" the event in which Candidate A trails behind B and C by one vote; "$B > A$" the event in which

Candidate A trails behind B by one vote with C more than one vote behind. Analogous notations are used to represent similar events. The table below represents the utility voter i would get with s_2 and s_{2A} after these events.

	s_2	s_{2A}		s_2	s_{2A}		s_2	s_{2A}		s_2	s_{2A}
ABC	v_i	1	AB	v_i	1	AC	$\frac{1}{2}$	1	BC	v_i	$\frac{v_i}{2}$
$A > BC$	$\frac{1+vi}{2}$	1	$AB > C$	v_i	1	$A > B$	$\frac{1+v_i}{2}$	1	$B > A$	v_i	$\frac{1+v_i}{2}$
$B > AC$	v_i	$\frac{1+vi}{2}$	$AC > B$	$\frac{1+v_i}{3}$	1	$A > C$	1	1	$C > A$	0	$\frac{1}{2}$
$C > AB$	$\frac{vi}{2}$	$\frac{1}{2}$	$BC > A$	v_i	$\frac{1+v_i}{3}$	$B > C$	1	1	$C > B$	$\frac{v_i}{2}$	0

When computing the expected value of $U_2 - U_{2A}$, all the terms corresponding with events not included in the table cancel out. On the other hand, it is obvious that all the differences between the utilities of events in the table lead to functions that are weakly increasing on v_i. Hence, $U_2 - U_{2A}$ is weakly increasing on v_i (if σ prescribes that everybody votes in Period 2, then s_2 and s_{2A} would coincide). Then,

$$U_2 - U_1 = (U_2 - U_{2A}) - (U_1 - U_{2A}),$$

which, as a function of v_i, is weakly increasing. Therefore, the larger v_i is, the larger is the incentive to vote in Period 2, which corresponds to point ii) in the statement. $\square$

When using the result above to conduct equilibrium analysis, we can rely on Lemma 2 to ascertain that only the second case in Proposition 1 is relevant under perfect Bayesian equilibrium.[10] This observation is summarized in the following corollary.

Corollary 1. *Suppose that all candidates are ex-ante symmetric and that there exists an* $\varepsilon > 0$ *such that the interval* $(1-\varepsilon, 1]$ *is contained in the support of distribution* F *from which* v_i *types are drawn. Then, all perfect Bayesian equilibria in weakly monotonic, anonymous, and symmetric strategies are in threshold strategies in which more partisan voters vote in Period 1 and less partisan voters cast their vote in Period 2.*

More precisely, there is a threshold $v^N \in [0, 1]$ *such that a voter* i *who attaches utility*

[10]Note that the existence of a perfect Bayesian equilibrium follows from standard arguments via the existence of a trembling-hand perfect equilibrium of the agent normal-form of the game. We conjecture also the existence of such an equilibrium in weakly monotonic and anonymous strategies, but can provide no proof.

$v_i < v^N$ to his intermediate candidate votes for his preferred candidate in the first period and each voter with $v_i > v^N$ votes in the second period.

The above results already capture some of the aspects that we believe will hold quite generally in sequential voting settings with strategic timing:

- In general, there will be no equilibria in which everybody votes in the same period.
- Equilibria will typically be in threshold strategies.
- More partisan voters tend to vote early to make their preferred candidate look stronger; less partisan voters tend to wait in order to make a more informed decision.

In general sequential voting settings there will be a large number of equilibria, since there are many ways to use the results of first period to coordinate on a candidate. A natural equilibrium (in the spirit of weak monotonicity) is that voters whose preferred candidate is (weakly) ahead after the first round will vote for her, and voters who have this leading candidate as their intermediate choice, with associated utility v_i sufficiently close to one, will also vote for her. Unfortunately, some issues even arise for such a natural idea. Suppose, for instance, that there are exactly two leading candidates with an equal number of votes after the first round. It will then be impossible for voters who have these two candidates as their first and second choice to coordinate on one of them (under symmetric and anonymous strategies). Yet, it may be possible to coordinate on the candidate who is behind (by only a few votes, say) for those voters who have this trailing candidate as their first or second choice.

In Sections 3 and 4, we present two particular cases of our general sequential voting setting in which equilibrium selection can be done under natural assumptions.

3 A model with ex-ante symmetric candidates

3.1 The model

We start with a simple and very stylized example. Consider a setting in which a voter either cares only about getting one particular candidate elected, being indifferent between the other two, or the voter cares only about preventing a certain candidate from winning, being indifferent between the other two. More precisely, we consider the extreme case where $v_i \in \{0, 1\}$, $v_i = 0$ represents "partisans" and $v_i = 1$ represents "averters". BC-voters are also called A-averters. Similarly, AC-voters and AB-voters are called B-averters and C-averters, respectively.

Compared to the situation described in Example 1, where we showed that, in general, fully responsive strategies are incompatible with equilibrium conditions, here AB-voters are indifferent between A and B, so the issues of the example do not arise. At the same time, AB-voters' indifference between A and B makes full responsiveness quite natural, since it merely requires that they vote for the "strongest" of the two in Period 2.

We assume that the probability of a voter being a partisan is $p \in [0, 1)$ and the probability of being an averter is $1-p$. Since there are three partisan types, we obtain from the symmetry assumption that the probability of each particular partisan type is $\frac{p}{3}$ and the probability of each of the three averter types is $\frac{1-p}{3}$.

Given a probability of $q \in [0, 1]$, let σ_q be the strategy profile defined as follows:

- Partisans vote for their preferred candidate in Period 1.[11]
- If $p > q$, averters vote for the leading candidate of their two preferred candidates in Period 2. If they tie, they randomize between them with equal probabilities.
- If $p \leq q$, an averter acts as before with a probability of $\frac{1-q}{1-p}$; with the remaining probability, $\frac{q-p}{1-p}$, he will vote in Period 1, randomizing between his two preferred candidates with equal probabilities.

Defined in this way, σ_q is a symmetric, fully responsive, and anonymous strategy profile. Moreover, it is worth noting that σ_q depends on p, the expected proportion of partisans in the model. Further, as long as $p \leq q$, the expected number of voters in Period 1 will be $p + (1-p)\frac{q-p}{1-p} = q$.

3.2 Theoretical results

Proposition 2. *In the ex-ante symmetric model, the following statements hold:*

[11] For the sake of completeness, off-path behavior is specified so that, conditional on the zero-probability event of not having voted in the first period, partisans vote for their preferred candidate in the second period.

i) For each number of voters N, there is $q^N \in [0,1]$ such that, for each expected proportion of partisans $p \in [0,1)$, strategy σ_{q^N} is a perfect Bayesian equilibrium.

ii) All symmetric perfect Bayesian equilibria in fully responsive and anonymous strategies are σ_q strategies.

Proof. Throughout the proof, when studying the incentives of an averter, we take, without loss of generality, a C-averter.

STATEMENT i). For the first part of the proof, we start checking the incentives of voter i when he knows that a strategy σ_q is being played, with $q \in [0,1]$. Recall that, by definition, σ_q is anonymous and fully responsive.

First-period incentives. If i is a partisan voter, by the ex-ante symmetry of the candidates and the full responsiveness of σ_q, in case of voting in Period 1, i should vote for his preferred candidate, as σ_q prescribes. Suppose now that voter i is a C-averter. Relying again on the ex-ante symmetry of candidates and the full responsiveness of σ_q, voter i is indifferent between voting for A or B (with both being preferred to C), so randomizing between them as σ_q prescribes is a best response.

Second-period incentives. If i is a partisan voter, the strategy specifies voting for his preferred candidate (even "off-path"), which is clearly optimal in any subgame in the second period. Suppose now that i is a C-averter. Consider a subgame in which A scored more votes than B in the first period. Let p_A and p_B denote the probability that, given the Period 1 score, a voter will vote for A and B, respectively. Due to the full responsiveness of σ_q, $p_A \geq p_B$. Thus, by Lemma 1, for voter i Events 1a, 1b, and 1c in the lemma are weakly more likely to occur than Events 2a, 2b, and 2c, respectively. Note that these are the only events in which switching his vote between A and B changes i's utility. Voting for A instead of B has the following implications in the above events: i) under Event 1a, it increases i's expected utility by $\frac{1}{2}$ while under 2a this utility is reduced by $\frac{1}{2}$, ii) under event 1b, it increases i's expected utility by $\frac{1}{3}$ and under 2b this utility is reduced by $\frac{1}{3}$, and iii) under event 1c, it increases i's expected utility by $\frac{1}{2}$ and under 2c this utility is reduced by $\frac{1}{2}$. Hence, Lemma 1 implies that voting for A is indeed optimal in these subgames.

Incentives across periods. To show that σ_q is indeed a perfect Bayesian equilibrium, we still need to show that the tradeoffs between voting in Period 1 and Period 2 are properly balanced. More precisely, we have to show that partisans are best responding by voting in Period 1 and, since averters may randomize between Period 1 and Period 2, we should show that, when doing so, they are indifferent between the two possibilities. The argument for partisans is straightforward - it simply relies again on the full responsiveness of σ_q. Concerning averters, it is not true that, for each $q \in [0, 1]$, they are indifferent between voting in Period 1 and Period 2 when playing according to σ_q. We show that there is $q^N \in [0, 1]$ such that this indifference holds. We distinguish two cases: $p = 0$ and $p \in (0, 1)$.

- **Pure averter population ($p = 0$).** Consider the incentives of a voter i who is a C-averter. Let u_1 denote i's expected utility if he votes in Period 1, with σ_q prescribing that he randomizes between A and B. Let u_2 denote i's expected utility if he votes in Period 2, with σ_q prescribing that he votes for the leading candidate from A and B (randomizing between them if they are tied). Since $p = 0$, according to σ_q, each averter will vote with a probability of q in Period 1 and with a probability of $(1 - q)$ in Period 2. Setting aside voter i's vote, the set of all possible scores after Period 1 is given by

$$\{(k_A, k_B, k_C) : \text{such that } k_A, k_B, k_C \in \{0, 1, 2, \ldots\} \text{ and } k_A + k_B + k_C \leq N - 1\}.$$

Given q, the probability of one such score (k_A, k_B, k_C) can be computed as

$$P_{k_A,k_B,k_C}(q) = \frac{(N-1)!}{k_A!k_B!k_C!\,(N-1-k_A-k_B-k_C)!} \left(\frac{q}{3}\right)^{k_A+k_B+k_C} (1-q)^{N-1-k_A-k_B-k_C}.$$

These probabilities are all continuous in q. Furthermore, given an intermediate score (k_A, k_B, k_C), we can calculate i's (conditional) expected utility of voting in Period 1 and in Period 2, denoted by $u_1(k_A, k_B, k_C)$ and $u_2(k_A, k_B, k_C)$, respectively. Hence, the ex-ante expected utility difference $u_1(q) - u_2(q)$ is also continuous in q.

For $q = 0$, almost surely nobody votes in period 1 and $P_{0,0,0} = 1$. Then it is clearly optimal for a C-averter i to vote in Period 1, since if he votes for candidate A, for instance, he will induce all C-averters and all B-averters to vote for A, which will reduce the probability of C winning the election. Hence $u_1(0) - u_2(0) > 0$.

For $q = 1$, it is clearly optimal for a C-averter to vote in Period 2. Since everybody votes in Period 1, i's vote in Period 1 will have no impact on the voting behavior of the remaining voters. Yet, informed voting in Period 2 can make a difference. Let us, for instance, consider of a situation in which $k_B = k_C > k_A + 1$, where voting for B is clearly better than voting for A. Hence, $u_2(1) - u_2(1) < 0$.

Therefore, the continuity in q implies there is at least one $q^N \in (0, 1)$ such that

$$u_1(q^N) - u_2(q^N) = 0.$$

- **Coexistence of partisans and averters ($p \in (0, 1)$).** For the incentives of an averter, it does not matter whether any other Period 1 voter is a partisan or an averter. By definition of σ_q, Period 1 voters independently vote for each candidate with a probability of $\frac{1}{3}$. Thus, the arguments from the case without partisans immediately generalize when the probability of partisans is $p \leq q^N$, where q^N is taken as the largest value for which $u_1(q) - u_2(q) = 0$ in the case a of pure averter population considered above. Then, in terms of incentives, partisans simply replace some of the averters who vote in Period 1. More precisely, if averters vote independently with a probability of $\frac{q^N - p}{1-p}$, then the probability that a randomly selected voter will vote in Period 1 is $p + (1 - p)\frac{q^N - p}{1-p} = q^N$ and averters are indifferent between voting in Period 1 and in Period 2.

  On the other hand, if $p > q^N$, continuity implies that, for each $q > q^N$, $u_1(q) - u_2(q) < 0$, since we selected q^N to be the largest value at which equality holds. Thus, averters have a strict incentive to vote in Period 2.

STATEMENT ii). Note that all σ_q strategies only differ with regard to the proportion of people voting within each period, but the voting behavior inside each period is the same.

Second-period behavior. It is obvious that the combination of ex-ante symmetry of the candidates, anonymity, and full responsiveness uniquely characterizes Period 2 behavior, for both partisans and averters, and that it coincides with that prescribed by all σ_q.

First-period behavior. Due to ex-ante symmetry, anonymity, and weak monotonicity, a first-period voter has a strict incentive to vote for a candidate that gives him a positive utility (he would increase the probability that this candidate will lead after the first period).

In case of an averter, anonymity implies that he should randomize between his two preferred candidates. Again, this behavior coincides with that prescribed by all σ_q.

Period selection. Clearly, partisans strictly prefer to vote in Period 1, since it increases the probability that their candidate will lead after the first period. Concerning averters, anonymity implies that the probability with which an A-averter votes in Period 1 coincides with that of a B-averter and a C-averter. Thus, once this probability is pinned down, we have completely specified a σ_q strategy. □

Note that, in Proposition 2, q^N depends on N only, being independent of p (although σ_q does depend on p). Importantly, uniqueness of q^N would imply uniqueness of the symmetric perfect Bayesian equilibria in fully responsive and anonymous strategies. Although we conjecture that the probability q^N is indeed unique (given N), and all our numerical analysis suggest that this is the case, we were unable to formally prove this. In Section 4, where we find similar results, we discuss in further detail the challenges of proving uniqueness due to the complexity of the combinatorial numbers involved in the computations of the functions $u_1(q)$ and $u_2(q)$.

3.3 Numerical results

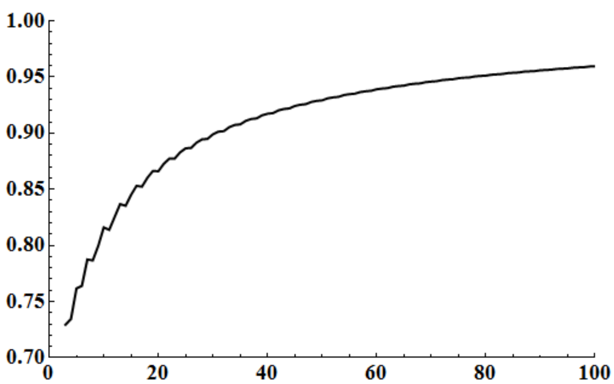
Although we were unable to obtain a general result for the uniqueness of the fully responsive equilibria described in Proposition 2, we could numerically solve for equilibrium for different population sizes. The numerical analysis suggests that the equilibrium is indeed unique but, maybe more importantly, also helps to draw some initial conclusions.

The aim of the numerical computations is to obtain the value of q^N that characterizes the proportion of people voting in Period 1 and in Period 2. To this end we work with the model with $p = 0$, i.e. there are no partisans. Since q^N is independent of p, this assumption is irrelevant for the qualitative implications of the analysis.

In Table 1 and Figure 1 we can see that the proportion of voters who cast their vote in Period 1 seems to approach one as the population size increases. In terms of voters' incentives, this means that the impact of "making your candidate look stronger" by voting in Period 1 outweighs that of "making a more informed decision and avoiding wasting your vote". Yet, although the probability of voting in Period 1 seems to converge to one, the expected number of voters also increases as the population size increases, albeit quite slowly.

Population	Proportion Period 1	Expected voters Period 2	Population	Proportion Period 1	Expected voters Period 2
N	q^N	$N(1-q^N)$	N	q^N	$N(1-q^N)$
5	0.762	1.192	55	0.935	3.592
10	0.816	1.840	60	0.939	3.655
15	0.845	2.326	65	0.942	3.749
20	0.866	2.678	70	0.946	3.787
25	0.887	2.837	75	0.949	3.829
30	0.899	3.032	80	0.951	3.902
35	0.908	3.226	85	0.954	3.931
40	0.917	3.308	90	0.956	3.961
45	0.924	3.410	95	0.958	4.021
50	0.929	3.539	100	0.960	4.043

Table 1: This table is a summary of the two plots.



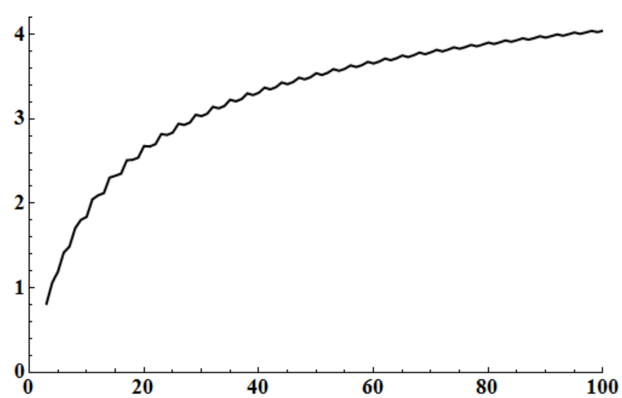


Figure 1: Both graphs have the population size on the x-axis. The left graph shows the probability of voting in Period 1; the right graph shows the expected number of voters in Period 2.

Interestingly, as can be seen in the plots, neither the expected proportion of people voting nor the expected number of voters are completely monotonic as functions of the population size.

Remark 1. It is worth explaining how the numerical analysis was conducted. We used Mathematica to find the probability q^N for the different values of N. To this end, we explicitly computed the probabilities of all the realizations of the electorate and used them to solve for the equilibrium point. Due to the combinatorial nature of the problem, the number of such realizations grows exponentially with N, as does the time required to complete computations. In particular, the cases with the largest values of N took several days.

4 A partially deterministic model

The model discussed in Section 3 was useful for capturing some qualitative features of the tradeoff between voting in Period 1 and voting in Period 2. However, it was not very useful for addressing the other key question of this paper: how is the likelihood a Condorcet loser winning affected by having a sequential election instead of a simultaneous one? In this section, we present a different simplification of the model that delivers results in this direction.

4.1 The model

Consider a scenario where we have only AB-voters, BA-voters, and C-partisans. The v_i values of AB and BA-voters are distributed independently and uniformly over the unit interval and the C-partisans have utility 0 for their intermediate candidate.[12] Moreover, in order to reduce the combinatorial difficulties arising in the model, we assume that, given size N of the electorate, the number of C-partisans, N_C, is fixed. In particular, we assume that $\frac{N+4}{3} < N_C < \frac{N}{2}$, which ensures that C is a Condorcet loser (less than 50% of voters support her in any pairwise comparison) but a certain amount of coordination is required to prevent her from winning.

[12]The assumption that voters' types are uniformly distributed is not crucial for the results, and can be replaced by any other continuous distribution with full support in $[0, 1]$. However, uniformity is convenient for both the formal analysis and the interpretation of the numerical results.

Before specifying how the remaining $N - N_C$ voters are split between AB and BA-voters, we will make some initial observations concerning the setting. Let $\bar{m} = N - 2N_C$. This value $\bar{m}$ is a threshold value in the sense that, if both A and B get more than $\bar{m}$ votes, coordination is not sufficient, and C will win the election if all C-partisans vote for candidate C:

- If candidate A gets more than $\bar{m}$ votes, then the number of votes B obtains is smaller than $N - N_C - \bar{m} = 2N_C - N_C = N_C$. Thus, C gets more votes than B.
- Analogously, if B gets more than $\bar{m}$ votes, then the number of votes A obtains is smaller than N_C. Thus, C gets more votes than A.

Therefore, to make sure that a certain amount of coordination is required for A or B to win, we assume that $N_{AB} > \bar{m}$ and $N_{BA} > \bar{m}$. More precisely, the number of N_{AB} and N_{BA} voters is obtained as follows. Let $M = N - 2(\bar{m} + 2) - N_C$. Note that $M > 0$, since

$$N - 2(\bar{m}+2) - N_C = N - 2(N - 2N_C + 2) - N_C = 3N_C - N - 4 > N_C + N + 4 - N - 4 - N_C = 0.$$

Let $d \in \{0, 1, \ldots, M\}$ be a number generated with a binomial distribution corresponding with the repetition of M Bernouilli trials with a probability of 0.5. Then $N_{AB} = \bar{m} + 2 + d$ and $N_{BA} = \bar{m} + 2 + M - d$. This ensures that the model has the following features: i) A and B are ex-ante symmetric, ii) a certain amount of coordination is required to win, and iii) a single voter is unable to induce effective coordination alone by making an informed decision in Period 2. In particular, under anonymous strategies, if there is a tie between A and B after Period 1, then C will almost surely win. This last observation also implies that, under anonymous strategies, C would almost surely win in a simultaneous election.[13]

Example 2. To fix ideas, let us consider the following numerical example. Let $N = 100$. Then N_C must be at least $104/3$, so N_C is at least 35. Suppose, indeed, that $N_C = 35$. Then

[13] In this footnote, we briefly relate our simultaneous election equilibria to those obtained in past works. In their setup, Myerson and Weber (1993) obtain three equilibria - two asymmetric equilibria in which these voters fully coordinate and one in mixed strategies in which coordination is only partial. Myatt (2007) finds partial coordination in a somewhat different model in which the distribution of voters' preferences is not common knowledge and voters have to estimate it from private signals. In our setting, given the information available to the voters, anonymity is a natural assumption which, moreover, is essential for our analysis of the equilibria under sequential voting. In contrast to the mixed equilibrium in Myerson and Weber (1993) we do not even have partial successful coordination in the simultaneous election in our setting. This is due to our assumption of a continuous distribution of types: only a zero probability mass of voters can be indifferent between voting strategies and, hence, willing to randomize.

$\bar{m} = 30$ and $N_{AB} \geq 32$ and $N_{BA} \geq 32$. In this case, $M = 1$ and only one voter will be chosen randomly to be AB or BA.

If candidate B gets $\bar{m} = 30$ votes and candidate C gets $N_C = 35$ at the end of the election, then A gets $N - \bar{m} - N_C = 35$, resulting in a tie between A and C.

Suppose that we are at an equilibrium in which C-partisans vote for C and that, after Period 1, there is a tie between A and B:

- If the tie came about with a number of votes exceeding $\bar{m}$, then C will win the election for sure.

- If the tie came about with a number of votes $m \leq \bar{m}$, then, under anonymity, AB-voters will vote for A in Period 2 and BA-voters will vote for B. Therefore even if one of these voters deviates to induce coordination between A and B, both candidates will win more than $\bar{m}$ votes, which means that C will win in any case. Note that we could still have that all the tied AB and BA-voters are pure averters ($v_i = 1$), in which case their randomization between A and B in Period 2 would have positive probability of leading to realizations beating C. However, the $v_i = 1$ situation has probability 0 and this is why we use the "almost sure" qualification in the discussion preceding this example. $\Diamond$

Before presenting the results related to this model, we will briefly discuss its strengths and limitations. The main simplifications are the restriction to just three types of voters and the partially deterministic nature of the model, under which N_C is fixed and N_{AB} and N_{AB} are known to be above a certain threshold. This is not only to make the model more tractable, but, since the setting ensures that there is no hope for A and B in a simultaneous election, we obtain an model in which we can cleanly measure the amount of coordination gained by the sequential election. Further, this approach can be seen as a natural approximation of what would occur in a large population model where the number of C-partisans is drawn from some distribution centered around a number N_C significantly larger than $\frac{N}{3}$ (and smaller than $\frac{N}{2}$) and then N_{AB} and N_{AB} voters are drawn with equal probabilities among the remaining $N - N_C$-voters. Our assumptions lead to a model where the cumbersome combinatorial aspects related to the set of potential realizations of the electorate are significantly reduced, which facilitates the theoretical and numerical analysis.

4.2 Theoretical results

First of all, note that Lemma 2 does not apply to this setting because we do not have ex-ante symmetry of the candidates and the types are not generated *i.i.d.*. In particular, there are equilibria in which everybody casts their vote in Period 1 and equilibria in which everybody votes in Period 2.

Similarly to what occurred with the ex-ante symmetric model in Section 3, compared to the situation described in Example 1, fully responsive strategies can be an equilibrium in this setting. The main reason for this is that a single voter is unable to induce effective coordination alone by making an informed decision in Period 2, which is what happened in Example 1. On the downside, this feature of the model also leads to equilibria in which everybody votes in Period 1. Moreover, one can easily define such equilibria to be in symmetric, fully responsive, and anonymous strategies. To illustrate this, let us consider the strategy profile in which AB-voters vote for A; BA-voters vote for B; and C-partisans randomize between A and B.

In order to keep the focus of the analysis on the tradeoff faced by AB and BA-voters when deciding in which period to vote and to eliminate unnatural equilibria such as that described above, we limit our attention to equilibria in which C-partisans always vote for C. We refer to these strategies as *C-strategies*. Note that this is merely a criterion for equilibrium selection, not a weakening of the equilibrium concept: in an equilibrium in C-strategies, C-partisans must respond optimally.

Proposition 3 below shows that, apart from the "pathological" equilibria in which everybody votes in Period 1, there are also equilibria in threshold strategies. It is worth noting that, qualitatively, the result is similar to Proposition 1 (which required ex-ante symmetry of the candidates). We need a final piece of notation. We claim that two strategy profiles are *realization equivalent* if they lead to the same distribution of probability over outcomes. In particular, given two realization equivalent profiles, all candidates will win the election with the same probability under each of these profiles.

Proposition 3. *In the partially deterministic model there exist symmetric perfect Bayesian equilibria in fully responsive and anonymous C-strategies. Any such equilibrium is realization equivalent to one of the following two:*

i) Everybody votes for his preferred candidate in Period 1.

ii) Given an AB or BA-voter i, there is a threshold $v^N \in (0,1)$ such that

- *If $v_i < v^N$, i votes for his preferred candidate in Period 1.*
- *If $v_i \geq v^N$, i votes in Period 2 for the candidate, A or B, who is ahead; if they are tied, i votes for his preferred candidate (randomizing if $v_i = 1$).*
- *C-partisans vote for C in Period 1.*

Proof. Since the statement is about realization equivalent strategy profiles, and types are drawn according to a continuous density defined on $[0,1]$, in most parts of the analysis there is no loss of generality in not explicitly considering AB and BA-voters with types $v_i = 0$ and $v_i = 1$.

Recall that, although we study strategy profiles in which C-partisans vote for C, we still have to show that doing so is indeed a best response. We start discussing the behavior in Period 2 after histories in which the election is not yet decided. For C-partisans in Period 2, voting for C is a weakly dominant action, so all C-strategies specify best responses for C-partisans in Period 2. Now consider the behavior of AB and BA-voters. First, full responsiveness requires that they vote for the leading candidate between A and B. Under these strategies, the lagging candidate has a zero probability of winning the election and, by construction, we know that a single deviation cannot change this; thus, AB and BA-voters best respond in Period 2. On the other hand, suppose that A and B are tied after Period 1. We claim that, under every equilibrium continuation in anonymous C-strategies, C will win for sure. Suppose, on the contrary, that the strategy at hand gives A or B a positive probability of winning the election. Then, by anonymity, A and B are equally likely to win. Given this, the unique best response of an AB-voter i with $v_i < 1$ is to vote for A (voters with $v_i = 1$ are indifferent). Similarly, BA-voters with $v_i < 1$ would vote for B. However, this contradicts that we are at an equilibrium in which A or B will win with a positive probability (recall that the model assumptions ensure that, without coordination, A and B will never win against C).

We have shown that, in Period 2, a) full responsiveness uniquely characterizes behavior after histories in which A and B are not tied and b) by anonymity, all potential equilibrium continuations after histories in which A and B are tied are realization equivalent to the one in the statement: C will win the election with a probability of one. Importantly, since the behavior of AB and BA-voters in Period 2 is independent of the behavior of C-partisans, given the number of votes received by A and B in Period 1, all fully responsive and anonymous C-strategies are realization equivalent (in particular, the proportion of C-partisans voting in each period is irrelevant).

We now turn to Period 1. Note that, by anonymity, if the strategies prescribe that all AB-voters vote in period t, then all BA-voters will also vote in Period t. Clearly, having all AB and BA-voters voting in Period 2 is not an equilibrium in fully responsive strategies. On the other hand, if they all vote in Period 1, anonymity implies that C will win the election for sure (the argument is analogous to the one used above for the case in which A and B were tied after Period 1). Hence, all equilibria in C-strategies in which all AB and BA-voters vote in Period 1 are realization equivalent to the equilibrium in point i) of the statement.

The rest of the proof deals with equilibria in which AB and BA-voters are split between periods 1 and 2. First, we show that a C-partisan cannot benefit from voting for A or B in Period 1 (in fact, anonymity implies that he would have to randomize between both candidates). This deviation would only be beneficial if it increased the probability of a tie between A and B after Period 1. However, it is clear that, given that strategies are anonymous, adding a vote for A or B would only reduce the probability of a tie.[14]

Next, we study the incentives of an AB-voter i, with v_i being his utility if B wins the election (incentives for BA-voters are analogous). Let q be the probability that an AB-voter votes in Period 1, and recall that we are in the case $0 < q < 1$. We now study the probabilities of those situations in which i would be pivotal. Let P_d^M denote the probability that, out of the M voters whose type is drawn using a Bernouilli distribution, d are of type AB. Then,

$$P_d^M = \frac{M!}{d!(M-d)!}\left(\frac{1}{2}\right)^d\left(\frac{1}{2}\right)^{M-d}.$$

For the sake of notation, in the equations below we use N_{AB^d} and N_{BA^d} to emphasize the dependence on d, i.e. $N_{AB^d} = \bar{m} + 2 + d$ and $N_{BA^d} = \bar{m} + 2 + M - d$.

We start with the situations in which i would prefer to vote in Period 1.

Case 1a. By voting in Period 1, voter i breaks a tie between A and B, where the tie came about with less than $\bar{m}$ votes. In this case, voting for A in Period 1 would lead to a

[14]Each vote cast in Period 1 has the same probability of being a vote for A as being a vote for B. Let k be the total number of such votes. Then ties are only possible if k is even. Under this condition, the probability of a tie clearly decreases in k.

victory for A and voting in Period 2 would lead to a victory for C. The probability of these ties is:

$$P_{1a}(q) = \sum_{d=0}^{M} P_d^M \sum_{k=0}^{\bar{m}-1} \frac{(N_{AB^d}-1)!}{k!(N_{AB^d}-1-k)!} q^k (1-q)^{N_{AB^d}-1-k} \frac{(N_{BA^d})!}{k!(N_{BA^d}-k)!} q^k (1-q)^{N_{BA^d}-k}.$$

Case 1b. By voting in Period 1, voter i breaks a tie between A and B, where the tie came about with $\bar{m}$ votes. In this case, voting in Period 1 for A would lead to a tie between A and C and voting in Period 2 would lead to a victory for C. The probability of these ties is:

$$P_{1b}(q) = \frac{(N_{AB^d}-1)!}{\bar{m}!(N_{AB^d}-1-\bar{m})!} q^{\bar{m}} (1-q)^{N_{AB^d}-1-\bar{m}} \frac{(N_{BA^d})!}{\bar{m}!(N_{BA^d}-\bar{m})!} q^{\bar{m}} (1-q)^{N_{BA^d}-\bar{m}}.$$

We now proceed to the situation in which i would prefer to vote in Period 2.

Case 2a. By voting in Period 1, voter i creates a tie between A and B, with less than $\bar{m}+1$ votes being cast. In this case, voting in Period 2 would lead to a victory for B and voting in Period 1 would lead to a victory for C. The probability of these ties is:

$$P_{2a}(q) = \sum_{d=0}^{M} P_d^M \sum_{k=1}^{\bar{m}} \frac{(N_{AB^d}-1)!}{(k-1)!(N_{AB^d}-k)!} q^{k-1} (1-q)^{N_{AB^d}-k} \frac{(N_{BA^d})!}{k!(N_{BA^d}-k)!} q^k (1-q)^{N_{BA^d}-k}.$$

Case 2b. By voting in Period 1, voter i creates a tie between A and B, with $\bar{m}+1$ votes being cast. In this case, voting in Period 2 would lead to a tie between B and C and voting in Period 1 would lead to a victory for C. If we let $L_{AB} = N_{AB^d} - 1 - \bar{m}$ and $L_{BA} = N_{BA^d} - \bar{m} - 1$, the probability of these ties is:

$$P_{2b}(q) = \sum_{d=0}^{M} P_d^M \frac{(N_{AB^d}-1)!}{\bar{m}!\, L_{AB}!} q^{\bar{m}} (1-q)^{L_{AB}} \frac{(N_{BA^d})!}{(\bar{m}+1)!\, L_{BA}!} q^{\bar{m}+1} (1-q)^{L_{BA}}.$$

Case 3a. By voting for A in Period 1, this candidate obtains $\bar{m}$ votes, while candidate B wins more than that. In this case, voting in Period 2 would lead to a victory for B and voting in Period 1 would lead to a tie between B and C. The probability of these ties is:

$$P_{3a}(q) = \sum_{d=0}^{M} P_d^M \frac{(N_{AB^d}-1)!}{(\bar{m}-1)!(N_{AB^d}-\bar{m})!} q^{\bar{m}-1} (1-q)^{N_{AB^d}-\bar{m}} \sum_{k=\bar{m}+1}^{N_{BA^d}} \frac{(N_{BA^d})!}{k!(N_{BA^d}-k)!} q^k (1-q)^{N_{BA^d}-k}.$$

Case 3b. By voting for A in Period 1, this candidate obtains to $\bar{m}+1$ votes, while candidate B wins more than that. In this case, voting in Period 2 would lead to a tie between B and C and voting in Period 1 would lead to a victory for C. The probability of these ties is:

$$P_{3b}(q) = \sum_{d=0}^{M} P_d^M \frac{(N_{AB^d}-1)!}{\bar{m}!(N_{AB^d}-1-\bar{m})!} q^{\bar{m}}(1-q)^{N_{AB^d}-1-\bar{m}} \sum_{k=\bar{m}+2}^{N_{BA^d}} \frac{(N_{BA^d})!}{k!(N_{BA^d}-k)!} q^k (1-q)^{N_{BA^d}-k}.$$

Then voter i's tradeoff between voting in Period 1 and voting in Period 2, which we denote by $T(q, v_i)$, is given by:

$$T(q, v_i) = P_{1a}(q) + \frac{1}{2}P_{1b}(q) - v_i\big(P_{2a}(q) + \frac{1}{2}P_{2b}(q)\big) - \frac{v_i}{2}\big(P_{3a}(q) + P_{3b}(q)\big).$$

Since $q \in (0,1)$, $T(q, v_i)$ is strictly decreasing in v_i, i.e. the less partisan i is, the smaller is his incentive to vote in Period 1. Importantly, this already implies that the equilibrium will be in threshold strategies, as point ii) in the statement claims. Moreover, since we assume that v_i types are generated by the uniform distribution, the probability of having a type smaller or equal to q is exactly q and, therefore, the set of interior equilibria is characterized by all the solutions of the equation $T(q,q) = 0$ (Figure 2 represents $T(q,q)$ for $N = 200$ and $N_C = 80$). To conclude the proof, we now merely need to show that at least one such solution exists.

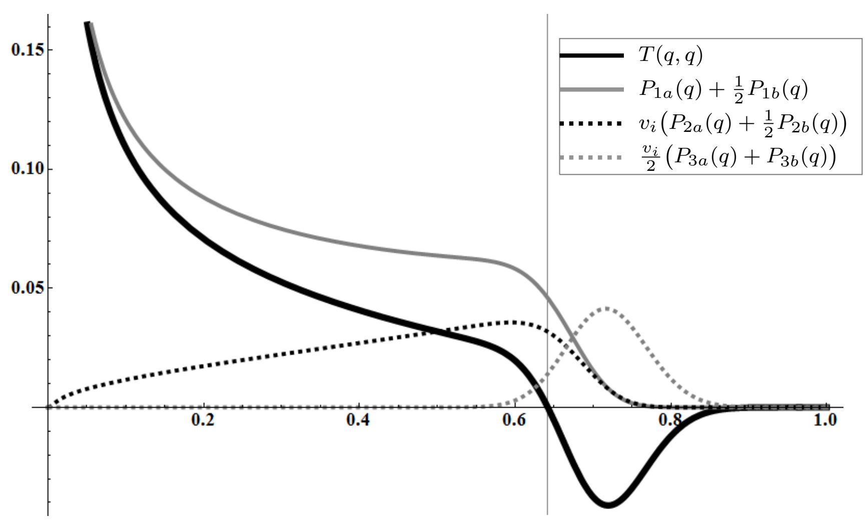


Figure 2: Illustrating the argument for the existence of equilibrium ($N = 200$, $N_C = 80$).

Clearly, as q approaches 0, the probability that an AB or BA-voter will vote vote in Period 1 approaches 0 and, hence, P_{1a} approaches 1, with all other probabilities vanishing, and so $\lim_{q\to 0} T(q,q) = 1$. On the other hand, as q approaches 1, the probability that all AB and all BA-voters will vote in Period 1 approaches 1. However, note that events leading to probabilities P_{1a}, P_{1b}, P_{2a}, and P_{2b} require that both A and B obtain a number of votes in Period 1 that is bounded away from N_{AB^d} and N_{BA^d}, respectively. Yet, $P_{3b}(q)$ only requires the number of votes for A to be bounded away from N_{AB^d} and, in fact, this number is $\bar{m}$, which is the highest number of votes obtained by A in any of the events leading to probabilities P_{1a}, P_{1b}, P_{2a}, and P_{2b}. Hence, as q approaches 1, the latter probabilities become arbitrarily less likely to occur than $P_{3b}(q)$ and, thus, $T(q,q)$ approaches 0 from below 0. Therefore, there has to be at least one value $q \in (0,1)$ for which $T(q,q) = 0$. $\square$

Ideally, we would have liked to prove the uniqueness of the equilibrium in threshold strategies whose existence is guaranteed by the above result, but we were unable to do so. One approach in order to obtain such a result could be to obtain some monotonicity result for $T(q,q)$. However, plots like the one in Figure 2, which vary significantly for different values of N_C, already suggest that this may be a hard task. Nonetheless, we conjecture that the equilibrium is indeed unique, and all our numerical analysis suggests that this is the case.

Concerning the nature of the equilibria studied in Proposition 3, it is relatively natural for equilibria in threshold strategies to be fully responsive. Ex ante, voters do not know whether $N_{AB} \geq N_{BA}$ or $N_{BA} \geq N_{AB}$. Now, if candidate A obtains more votes than candidate B, then it is most likely that $N_{AB} > N_{BA}$. Then, for those voters who were "undecided" enough to opt to wait for the second period, voting for the strongest of the two is a coherent choice.

4.3 Numerical results

As was the case with the symmetric model in Section 3, we where unable to obtain a general result for the uniqueness of the fully responsive equilibria described in the theoretical result, Proposition 3. However, we again managed to numerically solve for equilibrium.

Figure 3 summarizes the results when $N = 200$ and $N = 1000$. In the x-axis we have the proportion of C-partisans N_C/N. The y-axis represents probabilities. The black line represents the threshold value for N_{AB} and N_{BA} voters. Since types are drawn from the

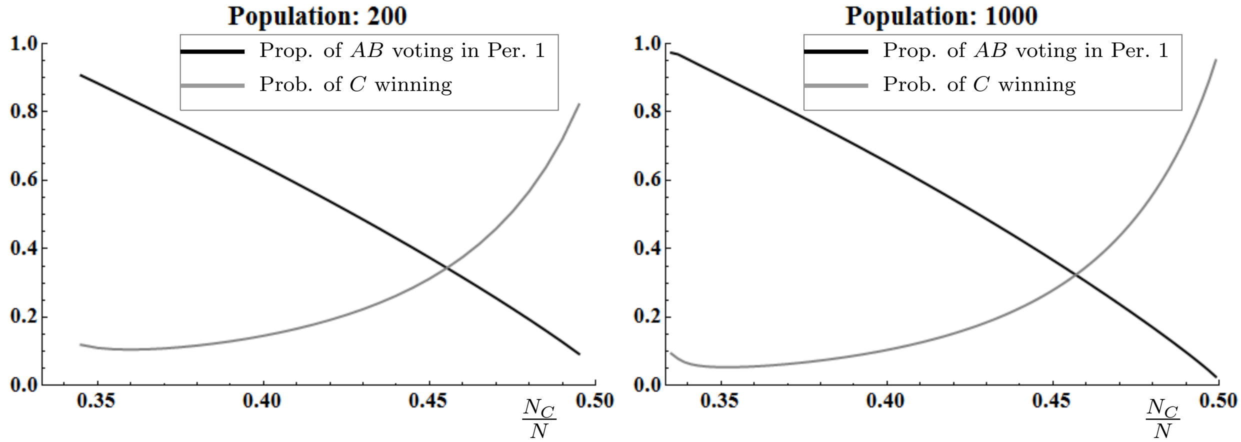


Figure 3: Threshold probabilities and likelihood of C winning the election as a function of the initial share of votes, $\frac{N_C}{N}$.

uniform distribution in $[0, 1]$, this coincides with the probability that an AB-voter will vote in Period 1. As we can see, the stronger candidate C is, the fewer people will vote in Period 1 (this is natural, since there would otherwise be insufficient number of people in Period 2 to beat C). The gray line represents the probability of C winning the election under the equilibrium in threshold strategies. Although this probability seems to grow as C becomes stronger, it is not completely monotone in N_C/N. When C is relatively weak (share close to 1/3), she may actually win more often if she is even weaker. It may be interesting to investigate this phenomenon further. Finally, it seems that the graphs are not very sensitive to population size.[15] This should come as no surprise, since our deterministic approach in a sense mimics the kind of realizations one would observe for very large N.

Remark 2. Again, the numerical analysis was conducted using Mathematica. First, to find the solutions of $T(q, q) = 0$ for the different values of N and N_C, and then, to compute the probability of C winning, we added the probabilities of all the realizations of the electorate under which C would win. Although the partially deterministic nature of the setting reduces the combinatorial complexity of the problem, the computing time also grows exponentially in N; the case $N = 1000$ took several hours, for example.

[15] Although we present solid lines, we only have a discrete set of points, since N_C takes only integer values. The larger N is, the more values we will have in the interval $(\frac{N+4}{3}, \frac{N}{2})$.

5 Discussion

We proposed and analyzed a new sequential voting system in which people may choose when to cast their vote and the intermediate score is announced after each period (or continuously). The theoretical analysis of this strategic sequential voting system is complex (something all papers on sequential elections must deal with) and we made a number of simplifying assumptions to gain progress and derive formal results. Nonetheless, we expect our key insights to hold beyond the specific assumptions: partisan voters, who care most about one candidate, have an incentive to vote early and averters, who care most about preventing a certain candidate from winning, tend to vote later after learning which other candidate has the best chances of competing successfully. This voting system may help to mitigate the hazard of a coordination failure that could make a Condorcet winner lose in simultaneous elections. It also allows voters to express the relative strength of preferences for the candidates to a certain degree, which may improve their welfare.

Our analysis suggests that this strategic sequential voting system has several benefits. We focused mainly on a comparison with simultaneous plurality voting. Comparisons to other voting systems, such as approval voting, may generate further insight.[16] Many other voting systems have been suggested in the literature, in fact too many to compare strategic sequential voting with each of them. Another issue is that real voters may not be completely rational, and they may be unable or unwilling to conduct the corresponding pivotal calculations.

As mentioned in the introduction and the body of the paper, our theoretical and numerical results deliver a good number of testable implications. Thus, it would be interesting for future work to conduct experiments to determine the extent to which they carry through to real experimental settings in a controlled environment. Similarly, one could also test these insights in a real-world setting (ideally, with clearly defined payoffs), starting, for example, with decisions made by relatively small committees or groups. Interestingly, several online "elections" already announce intermediate scores. This is the case, for instance, with some polls where the best player is voted in a competition or tournament. Contexts like this one can raise additional questions, such as the influence of this sequential voting system on voter turnout. On the one hand, the public announcement of intermediate scores may

[16] Refer to Brahms and Fishburn (1978), Myerson and Weber (1993), Weber (1995) and references therein.

create excitement and increase turnout. On the other hand, if it appears at some intermediate point as though the frontrunner cannot be beaten, voter turnout could fall. We did not consider costly voting and the impact of strategic sequential voting on turnout in such a context. These are relevant questions before implementing any new voting system in general elections, but the understanding of electoral turnout under costly voting is still very incomplete.[17] Even in simultaneous elections the question whether voter turnout is too low or too high from a welfare perspective turns out to be subtle.[18] We hope that this paper stimulates further theoretical, experimental, and empirical research on strategic sequential voting, leading to a solid understanding of the circumstances under which it is a good voting system.

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[17] See for example Feddersen and Sandroni (2006) for a summary of the difficulties standard voting models have in explaining voter turnout and an interesting new approach they suggest.

[18] See e.g. Börgers (2004) for a very natural model of costly voting with private values in which turnout is too high from a welfare perspective.

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