CHAPTER 3: CHILDREN'S UNDERSTANDING OF NUMBER PATTERNS

Anna S. Steinweg, IEEM University of Dortmund

One of the main aims of the maths 2000 project is to take both the child and the subject seriously (cf. Dewey, 1974). According to Devlin (cf. 1997), mathematics is seen as the science of patterns. Patterns can arise in geometrical themes as well as in arithmetical ones. The latter are called number patterns.

Looking through the maths 2000 textbooks (cf. Wittmann et al., 2000/01) one can find several examples of number pattern tasks offered to the students. The adaptation of mathematics as the science of patterns into textbooks is innovative for German schools and is not found in daily schoolwork. Reservations about patterns formed by 'pure' numbers are common. Mathematics is regarded as a boring subject and tasks with supposed connections to the child's daily life and its daily needs are believed as the appropriate approach. My dissertation research project (cf. Steinweg, 2001), which was inspired by the pattern in the maths 2000 textbooks and the special situation in German math lessons, focused on two major questions:

I How do children across ages react to the given number pattern, which are unfamiliar to them?
II To what extent can a kind of natural or genetic development of understanding of number patterns with advancing age be found?

Design and Theoretical Framework of the Research

The different tasks should offer children of all skill levels the opportunity to show their competence in dealing with number patterns. The themes polygonal numbers, sequences, number pairs with functional relation, number pyramids, and so called beautiful 'packages' were selected as tasks for the research because they show typical characteristics of mathematical patterns, and different appropriate approaches are possible. The various tasks can be divided into three types. Type A describes patterns formed by numbers, type B patterns formed by shape, which have to be recognised and continued. Type C are a series of calculation tasks (beautiful 'packages'), which are connected by patterns. One of the tasks in the 'package' does not fit into in the pattern and has to be replaced by a fitting one.

The understanding of number patterns is theoretically structured in three levels:

I Recognising and (intuitive) Continuing of the pattern
II Describing the pattern
III Explaining the pattern

These levels do not mirror a development schema but are understood as interdependent. They serve to both analyse the result and design the research. On the one hand the construct of levels allows different reactions of the students categorised qualitatively. On the other hand the research project has to be designed in such a way that the participating students have the opportunity to show their competencies of all

levels. In consequence two different designs were created. The first one to determine the level I – competencies of the children in a paper-and-pencil-test and the second one, an interview study, to offer the students the opportunity to show their competencies in all three levels.

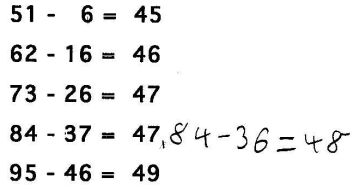
A sample of 257 students (111 female / 146 male) across ages (6/7 to 9/10 year–olds) of the German primary school was tested in the **paper-and-pencil-tests**. They came from three different schools with very different social backgrounds. Number patterns were not taught in any of the participating schools prior to the test. The test took place during the daily maths lessons. There was a limitation of time to one school lesson (45 min.). The students were free to do the tasks in their individual order.

In the **interview study** 60 children (34 female / 26 male) took part in the interview study, 15 of each age group (6/7, 7/8, 8/9, 9/10 year–olds). The interviews were held during school in a separate room. Each interview was videotaped. The interviews were designed for a 20 min. time period for the youngest ones (6 to 7 year–olds) and up to 45 min. for the oldest students (9 to 10 year–olds).

Analysis of the Children's Solutions

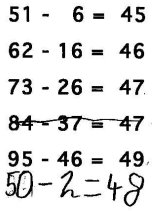
Every number pattern task could be solved in different mathematically correct ways. Nevertheless the solutions had to and could be categorised in an epistemologically evident way. Four categories were fixed, which will be described by examples of children's solutions to a 'beautiful package' (a type C – task for 2nd graders, 7 to 8 year–olds).

(1) *solved with the solution which suggests itself*



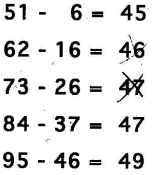
Kim replaces the fourth task by 84 – 36 = 48. She comments: "everywhere here is 6, 6, 6 ... and there is a 7."

(2) *solved with aspects of the pattern having been considered*



René tries to find the non-fitting task by searching the results. He recognises the double 47 and replaces the fourth task by 50 – 2 = 48.

(3) *solved without an apparent connection with the pattern*



Jana suspects the calculations to be wrong. She comments: "2 – 6 aren't 6 that are 0". First she crosses out the 46 than the 47 of the third task.

(4) *not worked on*

First the results of the paper-and-pencil-test and the interviews were quantitatively analysed whether the task was worked on or not. In the second step the solutions offered were categorised as shown above. The video documents were also qualitatively analysed with special emphasis on level II and III.

Results of the Research Studies

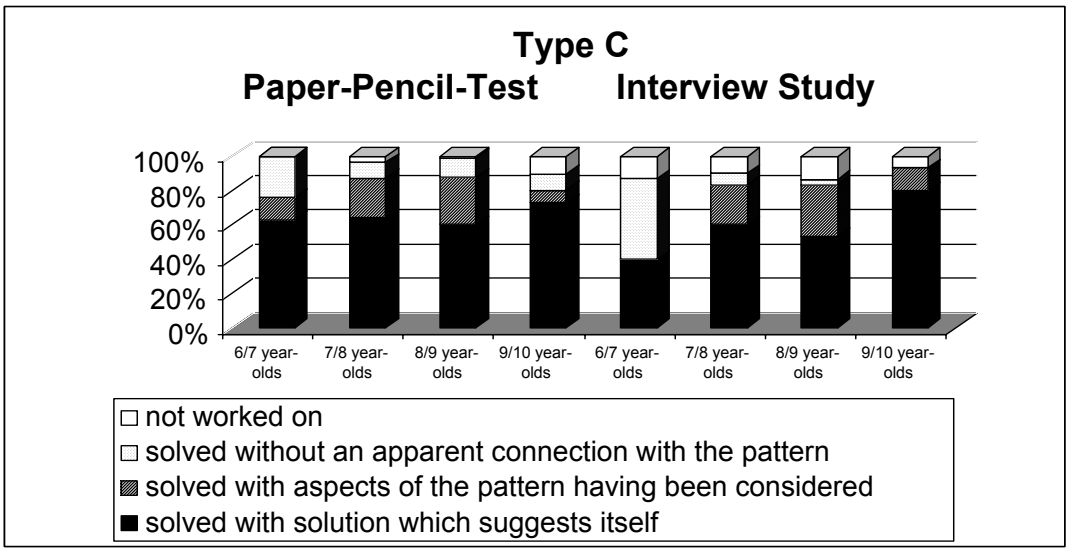
The main results of the **paper-and-pencil-tests** are in summary:

(1) The percentage of children whose answers belonged to category 1 or 2 averaged 60 % (i.e. the results of the type C – tasks see fig. below).
(2) Category 4 (not worked on) was more often found in solutions of older children.
(3) A genetic or continuous increase of category 1 and 2 solutions across ages could not be found.

The paper-and-pencil-test showed a great acceptance of number pattern tasks. The design of the following interview study allowed the children to show all three levels of understanding (recognising & continuing, describing, and explaining).

The main *quantitative results* of the **interview study** can be summarised as follows:

(1) The percentage of category 1 and 2 solutions varied from type to type. A continuous increase with advancing age could not be found (type C – results).
(2) The percentage of category 4 (not worked on) solutions showed no uniform tendency across ages and types.



The *qualitative results* have shown i.a. that

(1) the older children were more able to describe the pattern more generally. The younger ones often referred to examples in their descriptions.
(2) with advancing age the students were more able to recognise different patterns that simultaneously occurred in one complex structure, e.g. patterns of the sequence of addends and the one of the sums.
(3) calculation skills were useful in solving number pattern tasks.

(4) the children showed no spontaneous need to explain the structures and occurring patterns (level III).

Conclusions

The research project gained insight into the children's understanding of number patterns. The results should not be overestimated. The shown and assessed solutions only mirror the present strategy the individual student regarded suitable, and do not give any information about the ability to use alternatives (cf. Litherland, 1997, p. 13). As a tendency the results of the studies are evident, because "... every particular is also a sample of a larger class. In this sense, what has been learned about a particular can have relevance for the class to which it belongs" (Eisner, 1998, p. 103). The tendency "provides us with a guide, not a guarantee" (Eisner, 1998, p. 105).

The research gave answers to the two questions posed in the beginning.

I *How do children across ages react to the given number patterns, which are unfamiliar to them?*

The children do not have any reservations about the unfamiliar tasks. 60 to 88% of the solutions offered in the paper-and-pencil-test and 40 to 94% of the reactions in the interviews could be categorised as 1 or 2. The number patterns were fully accepted by the students and did not need any extrinsic motivation.

II *To what extent can a kind of natural or genetic development of understanding of number patterns with advancing age be found?*

A genetic development of understanding with advancing age could not be found on principle, even though some of the older children have shown a qualitatively better insight into complex structures and were more able to describe patterns generally. Other researches, i.e. the pattern researches in Leeds, came to similar results: "It could not be expected to occur 'naturally' in all classrooms, and would have to be taught more actively" (Threlfall 1999, p. 27). The understanding of number patterns has to be motivated by the teacher in school lessons. The active thinking approach, the SLEs (cf. Wittmann, Chapter 1 and Steinbring, Chapter 5), and the patterns offered in the textbooks of the maths 2000 project provide the settings in which the teacher and the children can unaffectedly access mathematics as the science of patterns.

References

Devlin, Keith (1997). Mathematics: The Science of Patterns. 2nd Printing. New York: Scientific American Library

Dewey, John (1974). The Child and the Curriculum and The School and Society. 12th edition. Chicago, London: The University of Chicago Press

Eisner, Elliot W. (1998). The Enlightened Eye: Qualitative Inquiry and Enhancement of Educational Practice. Upper Saddle River (New Jersey), Columbus (Ohio): Prentice Hall

Litherland, Barry (1997). "Reflections on the Assessment of Mathematics" In: MT 159: 10 - 14

Threlfall, John (1999). "Repeating Patterns in the Early Primary Years" In: Pattern in the Teaching and Learning of Mathematics. Anthony Orton (Editor). London, New York: Cassell: 18 – 30

Steinweg, Anna S. (to appear 2001). Das Zahlenmusterverständnis von Kindern: Epistemologisch-pädagogische Grundlegung. Münster: LIT
Wittmann et al. (2000/2001). Das Zahlenbuch. Mathematik für das 1. – 4. Schuljahr. Leipzig, Düsseldorf, Stuttgart: Klett